# Paradoxes of special relativity

Today we are turning from metaphysics to physics. As we'll see, certain paradoxes about the nature of space and time result not from philosophical speculation, but from theories constructed in the physical sciences in response to experimental data.

This week, we will be talking briefly about two of our most fundamental, and well-confirmed, theories of the physical world: the special theory of relativity and quantum mechanics.

Given this topic, the presentation of the theories will be, in a mathematical sense, pretty superficial. The point will just be to present enough material for you to understand why the theories seem to lead to the paradoxes they do. The readings linked from the course web page go into more depth for those who would like to understand more of the science.

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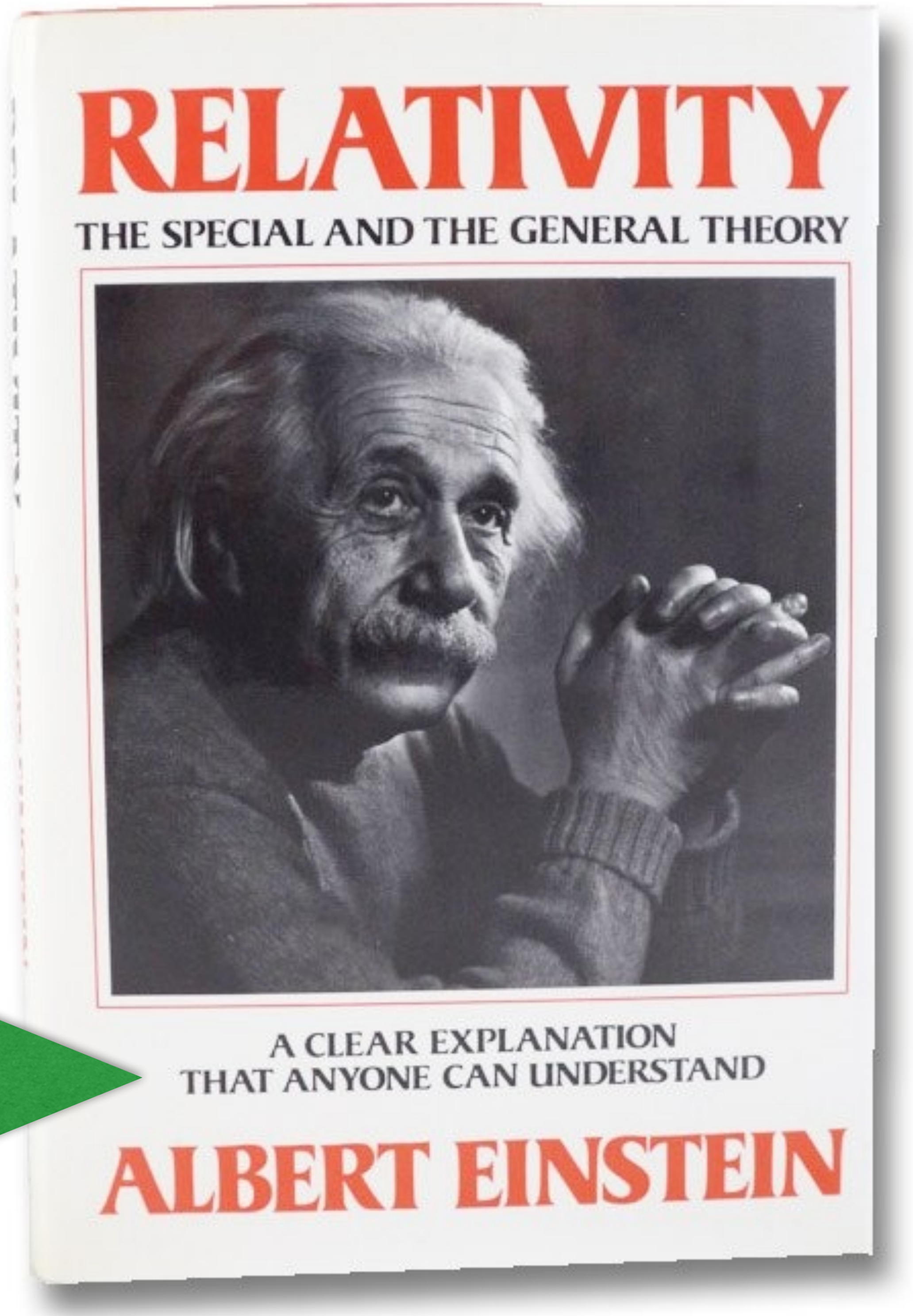
You may be somewhat daunted by the task of coming to understand the theory of relativity in one 50 minute philosophy class. And it is true that we will only cover the basics. But you should take heart from the subtitle of Einstein's book from which today's reading was taken.



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Einstein's theory is, for our purposes, an especially interesting one, because one can think of it as having its origins in a kind of paradox. (Einstein himself presents it that way in the reading on the web site.)

This paradoxes arises from the following three plausible, but jointly inconsistent, claims:

**Galilean relativity**: for any two objects moving at any speeds, their speeds relative to each other is the difference between their speeds if they're moving in the same direction, and the sum of their speeds if they are moving in the opposite direction.

**The speed of light is a law of nature.** (We'll follow convention by referring to this speed as "c".)

**The principle of relativity**: the laws of nature are the same in distinct frames of reference.

Each of these claims seems quite plausible on its own. But, as Einstein points out, they can't all be true.

If a ray of light be sent along the embankment, we see from the above that the tip of the ray will be transmitted with the velocity $c$ relative to the embankment. Now let us suppose that our railway carriage is again travelling along the railway lines with the velocity $v$, and that its direction is the same as that of the ray of light, but its velocity of course much less. Let us inquire about the velocity of propagation of the ray of light relative to the carriage. It is obvious that we can here apply the consideration of the previous section, since the ray of light plays the part of the man walking along relatively to the carriage. The velocity $W$ of the man relative to the embankment is here replaced by the velocity of light relative to the embankment. $w$ is the required velocity of light with respect to the carriage, and we have

$$w = c - v.$$

The velocity of propagation of a ray of light relative to the carriage thus comes out smaller than $c$.

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Here Einstein is carrying out a kind of thought experiment.

Imagine that the guy is walking at speed *v* and the light is propagating at speed *c*. How does this situation bring out the contradiction between the three theses above?

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The contradiction is perhaps more obvious when we imagine the person walking in the direction opposite the propagation of the light.

Now how fast is the light going relative to our walker, if Galilean relativity is true?

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So we can't keep all three of the above claims.

An initially plausible suggestion is that we should reject the claim that the speed of light is a law of nature, and say that the speed of light, like the speed of other things, can differ depending on one's speed relative to the light. But experiments designed to detect such differences in the speed of light failed to do so.

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However, this idea has some surprising consequences, which can be illustrated by example. (The example I use follows one Einstein also used in presenting his theory.)

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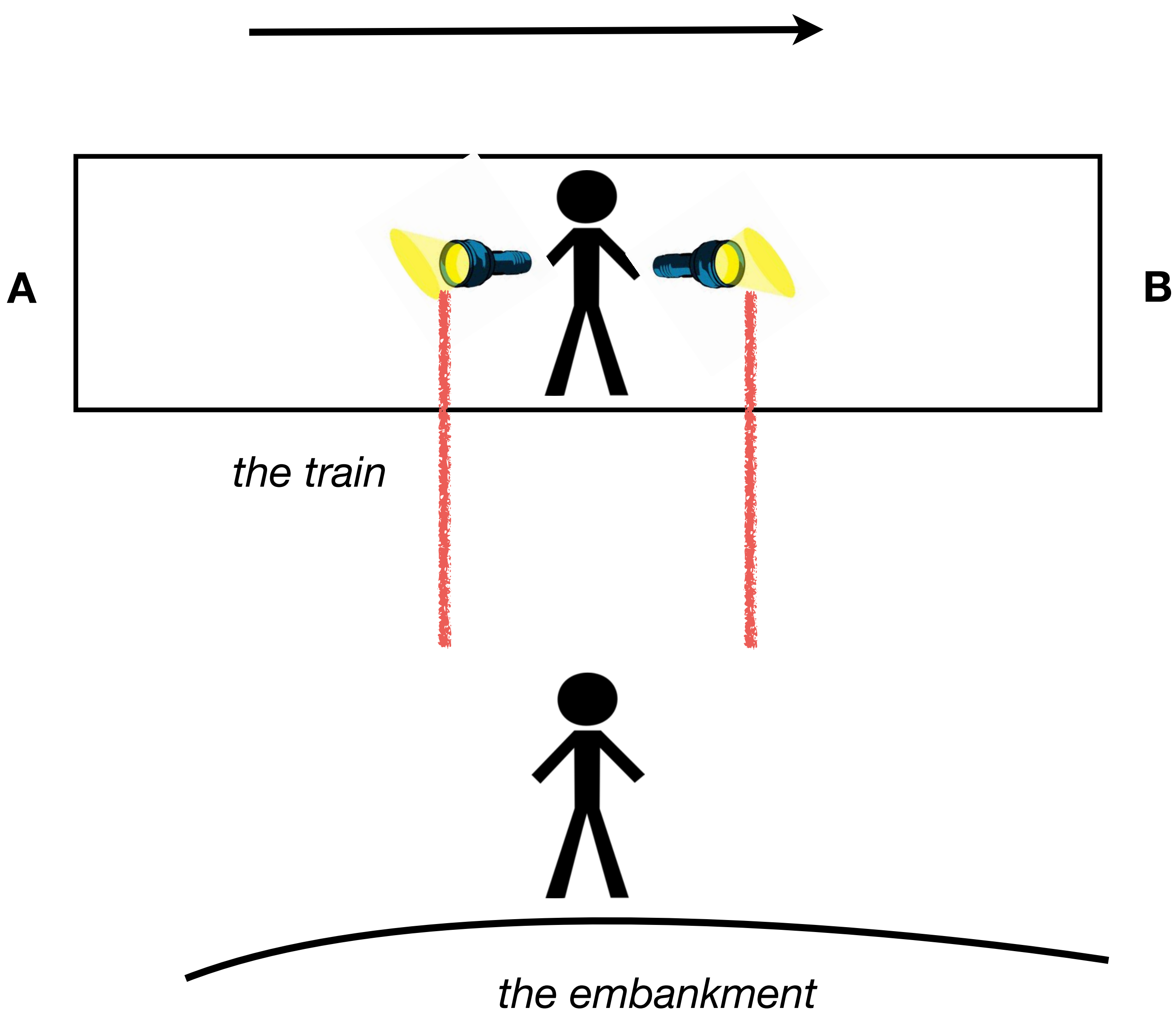


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Imagine two people, one in a train moving at a constant speed from left to right, and one on an embankment watching the train go by. We can imagine that the train is made of glass, so that the person on the embankment can see in.

Now imagine that the person in the train car simultaneously turns on flashlights pointed at the two walls of the train car, A and B; and imagine further that he's at the exact midpoint of the train car.

Think about this situation first from the perspective of the person in the train car. Does the light reach A or B first?

But now think about this from the perspective of the person outside the train car. Do we get the same result?

To answer this, let's mark the locations from which the two beams of light start.

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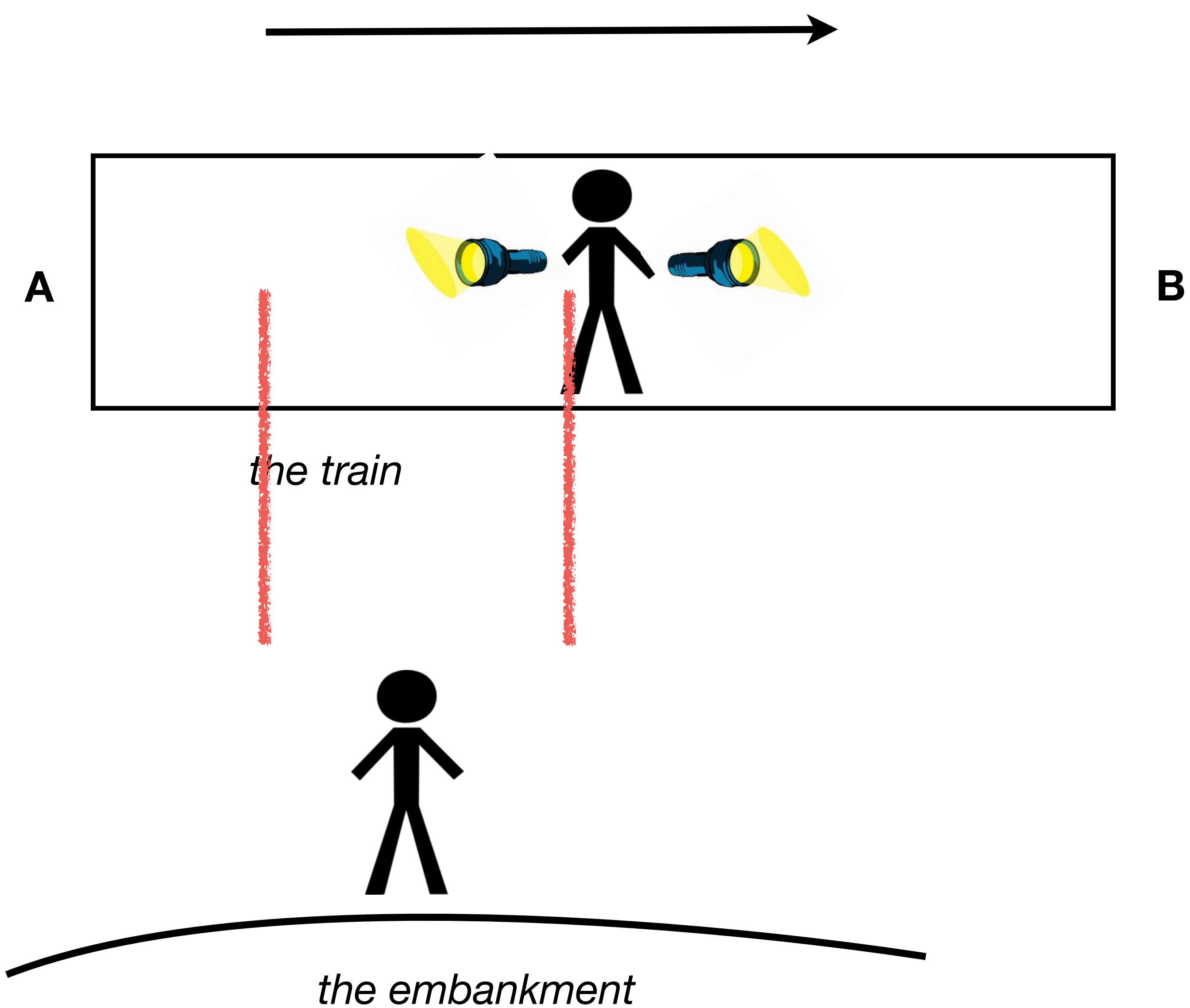


But now think about this from the perspective of the person outside the train car. Do we get the same result?

To see why not, it is important to note that the light takes some time to travel from the flashlights to the walls of the train, during which time the train travels some distance.

Hence it seems, looked at from the point of view of the person on the embankment, the location at which the left flashlight was turned on was closer to the location at which the light hits A than the location at which the right flashlight was turned on is to the location at which the light hits B.

But, given that the speed of both beams of light is the same from every frame of reference — including the person on the embankment — it follows that from his point of view the light hits A before it hits B. And this is not an illusion, if the speed of light is genuinely constant between frames of reference.

Hence, it seems, the light's hitting A is simultaneous with its hitting B relative to the frame of reference of the train, but not relative to the embankment.

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If simultaneity is relative to a frame of reference, so is **duration**. Consider the time between the flashlight being turned on and the beam of light hitting the back wall of the train car. This journey of the beam of light takes longer relative to the train car's frame of reference than relative to the frame of reference of the observer outside the train car.

The ordering of events can also change. Can you think of a variant of the above case in which one event happens before another from the perspective of the person on the train, but the ordering is reversed from the perspective of the frame of reference outside the train?

This is an extremely surprising result. We are accustomed to distinguish between facts which are dependent on a frame of reference or perspective, and facts which are not so dependent. We think of 'A is to the left of B' as in the first category, and 'A has more mass than B' as in the second category.

One would have thought of 'A happened before B' as also in the second category. But if Einstein's theory is true, this is simply a mistake.

But the relativity of simultaneity (and the consequent relativity of time ordering) is, while surprising, not exactly paradoxical. We turn now to three paradoxes of special relativity: the twin paradox, the grandfather paradox, and an apparently paradoxical result about the nature of the present, past, and future.

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The way into the first of these is via the phenomenon of **time dilation**. Intuitively, this is the phenomenon that if you are moving at a constant rate with respect to some frame of reference, time "speeds up" for you (slows down for them).

To see this, imagine the person on the train using a mirror opposite him in the car as a timekeeping device, which keeps time by the amount of time taken for light to reflect off of that mirror and back to him. Imagine again someone standing outside the train. Relative to someone outside the train, the light will be traveling further than for the person inside the train and hence (given the constancy of the speed of light) will take more time relative to the frame of reference outside the train.

The effect is that the "clock" constructed by the person inside the train will appear to be running slow. When their clock says that one second has passed, more than one second will have passed from the perspective of the frame of reference outside the train.

But now suppose that the person outside the train has their own clock, of the same general sort. From the perspective of the person inside the train, will that clock be running slow, or fast?

This is a surprising result. One thinks that if A's clock is running fast relative to B's, then B's clock must be running slow relative to A's.

In fact, one might think that this is more than surprising; one might think that it is contradictory. After all, what would happen if A and B got together and compared watches? Surely each could not find that the other's watch was slow relative to their own.

This is a simple version of the **Twin Paradox**, so called because the classic version of the paradox imagines two twins setting off in rockets going in opposite directions: it seems that it would be true for each to say that she is aging more quickly than her twin. But surely two people can't each be older than each other!

How this seeming paradox shows that the restriction to frames of reference in constant motion (neither accelerating or decelerating) is necessary.

Time dilation, along with other aspects of the theories of special and general relativity which go well beyond anything I have presented, has convinced many that if these theories are true, then time travel should be possible. The possibility of time travel into the past gives rise to some puzzling questions.

Some of the hardest are brought out by the **grandfather paradox.** Suppose that you could travel back in time. Then presumably you could travel back in time to some point during the life of your grandfather. And presumably, if you were so inclined, you could kill your grandfather. But, presuming that you visited your grandfather at a time in his life prior to the conception of your parents, your so doing would prevent your being born. But then you would not have gone back in time to kill your grandfather.

We can lay this out explicitly as an argument, as follows:

1. Time travel is possible.
2. If time travel is possible, I can travel back in time and kill my grandfather in 1920.
3. I can travel back in time and kill my grandfather in 1920. (1,2)
4. If I kill my grandfather in 1920, my grandfather dies in 1920.
5. If my grandfather dies in 1920, I am never born.
6. I can travel back in time and bring it about that I am never born. (3,4,5)
7. If I am never born, I can never time travel.
8. I can travel back in time and bring it about that I never time travel. (6,7)

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C. Possibly, I both travel back in time and never time travel. (8)

What does all of this have to do with the TV show *Lost*?

More seriously, the puzzle here is a conflict between logical constraints on what time travelers could and could not do and our intuitive view of our own freedom of the will.

The tempting idea is that **if** we could go back in time, then surely we would then as now be free to do what we want; and surely this means that it is genuinely possible for us to do things we have the opportunity to do, such as killing our former selves. But this is **not** possible; hence either time travel must not be possible, or there would be some sort of odd asymmetry between our free will now and our free will post-time travel, or our views about the nature of our freedom of the will must be mistaken.

But perhaps the deepest philosophical puzzles raised by the theory of relativity involves our intuitive distinctions between the past, the present, and the future.

Many of us are inclined to endorse claims like these:

> What is past is no more; what is future is yet to be; only the present is real.
>
> Dinosaurs do not exist. They used to exist; but that was in the past.
>
> Notre Dame's 12th national championship does not exist. It will exist; but it does not exist yet.

All of these claims seem to make use of some fundamental distinction between the status of present things and events, on the one hand, and past and future things and events, on the other.

But it can seem a bit difficult to make sense of the distinctions between past, present, and future, if Einstein's theory is correct. For it seems like 'past' is equivalent to 'earlier than now.' But we have already seen that A can be earlier than B from one frame of reference, but B earlier than A from another. The same line of reasoning can be used to show that A can be past from one frame of reference, but present from another frame of reference. Can you think of a way of adapting the example of the flashlights and the train to demonstrate this?

Does this cast doubt on the sorts of commonsense claims listed above? Should it change our view of the relative reality of past, present, and future?