Generalized quantifiers

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1 The problem posed by NP's within NP's

Our previous semantics for quantified NP's does not, in a way, assign any entities as the semantic values of those NP's. Consider, for example, 'every student.' On that semantics, we are told that such an NP combines with a sentence S to form a sentence, and that that sentence is true with respect to an assignment g iff

for all x, if $x \in [\![\text{student}]\!]$, then $[\![\text{S}]\!]^{g[x/e1]}$=1

Thus we don't get a semantic value assigned direct to the NP, in the way that we do for a name or a VP. Instead, we get a rule for determining the truth of a whole sentence composed of a sentence combined with the relevant NP.

This approach is fine so far as it goes. The problem is that it does not go quite far enough, because not all quantified NP's can be understood as combining with sentences to form sentences. A good example of this is NP's embedded within other NP's, as in

Every man next to a cat is unlucky.

Here 'a cat' can't be understood in any obvious way as combining with a sentence to form a sentence. Similar problems are posed by

Every man and some dog snores.

especially when we recall that we can't always 'analyze away' NP conjunction in favor of sentence conjunction when dealing with collective verbs.

We could, of course, add to our lexicon entries for each individual complex NP; so there could be a lexical entry which said

> $[\![$Every β next to a S$]\!]$=1 iff ...

What would be the problem with this?

2 NP's as denoting sets of sets

In response to this problem, it seems as though we need to modify our semantics for quantified NP's to assign them semantic values from which we can derive the semantic values of complex NP's in a compositional way.

The basic idea sketched in Ch. 9 of the text about how to do this is best illustrated by example. Consider the NP 'every man.' The suggestion is that the semantic value of this NP is a set of sets: it is the set of all of those sets which are such that every man is a member of them. So the members of $[\![$every man$]\!]$ will include $[\![$man$]\!]$, but also the set which contains every member of $[\![$man$]\!]$ and, in addition, contains Stanford Hall. Indeed, $[\![$every man$]\!]$ will include every set which is obtained by 'starting off with' $\{x$:x is a man$\}$ and adding some more stuff to it.

By analogy, $[\![$some man$]\!]$ will be the set of all those sets whose members include at least one man.

Now consider how we might use this assignment of semantic values to give us truth conditions for

> Every man snores.
> Some man snores.

In a way, our treatment of these sentences is the reverse of our usual treatment of

> Jack snores.

which we take to be true iff $[\![$Jack$]\!] \in [\![$snores$]\!]$. This won't work for our sentences involving NP's; it will never be true that $[\![$every man$]\!] \in [\![$snores$]\!]$, since sets don't snore. Instead we reverse the treatment, and regard the first sentence above as true if $[\![$snores$]\!] \in [\![$every man$]\!]$. Can you see why this, intuitively, gives us the right results?

One way to look at this is to think about 'Jack snores' as saying something about Jack – namely, that he snores – and to think of 'Every man snores' as saying something about snoring – namely that it is done by every man.

We can write out the semantic values of NP's using the following notation:

$$[\![\text{every man}]\!] = \{X \subseteq U\colon [\![\text{man}]\!] \subseteq X\}$$

I.e.: the set which has as its members every subset X of U (the domain) of which ⟦man⟧ is a subset – of which every man is a member.

How might you extend this to 'most men'? The semantic value of this will be the set of all those sets which contain most of the men – i.e., the set of all those sets which are such that their intersection with ⟦man⟧ is bigger (has more members) than the intersection of their complement with ⟦man⟧.

Of course, we can't just assign semantic values to phrases like 'most men' – we have to derive these compositionally. The idea, roughly, is that (as before) the semantic value of a common noun will be a set of individuals – and we now take the semantic value of a determiner, like 'every' or 'most' or 'the' or 'some' – to be a function from sets of individuals to the relevant sets of sets.

Which function would 'the' express?

This gives us the materials to solve our puzzle about sentences like 'Every man next to a cat is unlucky.' We might treat this in the way we treated relative clauses before, on which ⟦man⟧ and ⟦next to a cat⟧ are both sets of individuals, and ⟦man next to a cat⟧ is their intersection. The question is how we derive ⟦next to a cat⟧. It was a little hard to see how this would work on our former view of 'a cat', on which it combined with sentences. But now that we know that ⟦a cat⟧ will be the set of sets which have at least one cat as a member, we can begin to see how, with a treatment of prepositions, we might derive the result that ⟦next to a cat⟧ is the set of individuals that are next to at least one cat.

3 NP conjunction and disjunction

So far we've provided treatments of sentence conjunction and disjunction, and VP conjunction and disjunction. But we've been so far unable to arrive at a satisfactory treatment of NP conjunction and disjunction. Generalized quantifiers provide a natural way to do this.

Consider

> Every man and some dog snored.

Now that we have sets assigned as the semantic values of NP's, it is natural to take conjunction between NP's to signify the intersection of those sets (and to take disjunction to signify union).

We know that ⟦every man⟧ is the set of all sets which have ⟦man⟧ as a subset, and ⟦some dog⟧ is the set of all sets which have a non-empty intersection with ⟦dog⟧. If we take the union of those sets, we get the set of all sets which have among their members every man, and at least one dog. Applying our semantic above to this sentence, we get the result that it is true iff the set of things that snore is a subset of this set. And this seems to be exactly what we want.

But now consider

> Every man and Loren snored.

It seems as though we should be able to treat this sentence in a way parallel to the way that we treated the one above. But since $[\![\text{Loren}]\!]$ is not a set, there's no obvious way to apply the treatment of conjunction as expressing intersection.

(One might think: we could just treat $[\![\text{Loren}]\!]$ as {Loren}. What would be the result of combining this view with the view that 'and' expresses intersection?)

To solve this problem, we can treat names as generalized quantifiers as well. On this view, we discard our previous view that $[\![\text{Loren}]\!] = \text{Loren}$, and instead take $[\![\text{Loren}]\!]$ to be the set of all sets that include Loren. Applying this to the above sentence seems to give us just the right result.

This also helps with a puzzle we discussed in connection with plurals. Recall our sentence

> Bob, Jim, and Kate gathered by the window.

Here we noted that 'and' seemed to be functioning as a device to express union rather than intersection, which is very puzzling. But on our present view of names the right treatment of this sentence falls out pretty naturally (pending a satisfactory treatment of collective verbs). On this view, $[\![\text{Bob, Jim, and Kate}]\!]$ will be the set of all those sets which include Bob and Jim and Kate.

4 Negative polarity items

NPI's are so-called because they can occur grammatically in certain 'negative contexts', but not in other, apparently quite similar contexts. Examples include 'ever' and 'any.' Consider:

> No students ever read books.
> * Many students ever read books.

One of the central problems in understanding such expressions is to understand what a 'negative context' is. Such contexts don't, after all, always include expressions for negation. In fact, when we consider examples, the relevant distinction between contexts can seem a bit elusive:

> * Every student ever read books.
> Every student who ever read books was smart.
> Few students ever read books.
> Few students who ever read books are happy.
> Exactly two students ever read books.
> * At least two students ever read books.

Here's one way to capture this distinction in terms of our semantics for generalized quantifiers. Consider some sentence (like the above) formed from a Det, and N_c, and a VP. Both Nc's and VPs will have sets of individuals as their semantic values. So, for each occurrence of an N_c or VP, we can ask ourselves: does that sentence entail any sentence obtained by replacing that N_c or VP with an expression whose semantic value is a subset of that N_c or VP? If so, then we say that the context in which that term occurs is a *downward monotone context.*

By contrast, if the reverse direction of entailment holds, and the relevant sentence entails any sentence obtained by replacing the N_c or VP with one whose semantic value is a *superset* of the original, we say that it is an *upward monotone context.*

The hypothesis then is that NPI's can occur in a context iff it is downward monotone. Can you see how to apply this to the pairs

> No student ever read books.
> * Some student ever read books.

> Every student who ever read books was smart.
> * Some student who ever read books was smart.

This is interesting, because it appears to show that what looks at first like a mere linguistic oddity is well-correlated with certain features of the semantics we have been developing.